32.22 **$195°F$ water flows through an insulated copper pipe in a $60°F$ room. The pipe is nominal $3in$ diameter (I.D. $3.068in$, O.D. $3.5in$) and is wrapped in $1in$ thick insulation. The thermal conductivities of the pipe and the insulation are $239\frac{Btu}{hr \cdot ft \cdot °F}$ and $.03\frac{Btu}{hr \cdot ft \cdot °F}$, respectively. The film coefficient outside the insulation is $1.5\frac{Btu}{hr \cdot ft^2 \cdot °F}$. There is a contact resistance of $0.05\frac{hr \cdot ft^2 \cdot °F}{Btu}$ between the insulation and the pipe. What is the heat loss from the insulated pipe per linear foot?**

A. $4\frac{Btu}{hr \cdot ft}$

B. $15\frac{Btu}{hr \cdot ft}$

C. $23\frac{Btu}{hr \cdot ft}$

D. $46\frac{Btu}{hr \cdot ft}$

Draw the cross section of the pipe and label the dimensions.

Write an expression for the total resistance through all layers including the pipe wall, insulation, contact surfaces, and films. Working from the inside out, start by assuming that the inner pipe surface is at equilibrium with the $195°F$ flowing water inside the pipe.

Next, consider conduction through the pipe wall. Because steel pipe is a good conductor and a poor insulator, it is valid to assume there is negligible resistance from the pipe wall. Since the thermal conductivity has been provided, this can be shown quantitatively for reassurance. Use the Cylindrical Wall Conduction Resistance equation. Select a $1ft$ section of pipe since the question ultimately asks about heat loss *per foot*.

$$R_{pipe} = \frac{ln\left(\frac{r_2}{r_1}\right)}{2\pi kL}$$

$$R_{pipe} = \frac{ln\left(\frac{1.75in}{1.534in}\right)}{2\pi\left(239\frac{Btu}{hr \cdot ft \cdot °F}\right)(1ft)} = 8.8 \times 10^{-5}\frac{hr \cdot °F}{Btu} \approx 0$$

The first meaningful resistance encoutered is the contact resistance between the pipe wall and the insulation. Since there is no thickness associated with contact resistance, and looking at the units given, treat it as a film, where $r \propto \frac{1}{h}$. The area is the surface area of the outside of the pipe, given by $A_s = \pi DL$. Again, select a $1ft$ section of pipe for the analysis.

$$R_{contact} = \frac{1}{hA} = \frac{r}{A}$$

$$R_{contact} = \frac{0.05\frac{hr \cdot ft^2 \cdot °F}{Btu}}{\pi\left(\frac{3.5}{12}ft\right)(1ft)} = 0.055\frac{hr \cdot °F}{Btu}$$

Next, determine the thermal resistance provided by the insulation, which is based on the cylindrical wall conduction resistance equation previously used. Note the outer and inner radii refer to the dimensions for the insulation and not the pipe.

$$R_{insulation} = \frac{ln\left(\frac{r_2}{r_1}\right)}{2\pi kL}$$

$$R_{insulation} = \frac{ln\left(\frac{2.75in}{1.75in}\right)}{2\pi\left(0.03\frac{Btu}{hr\cdot ft\cdot{}^\circ F}\right)(1ft)} = 2.398\frac{hr\cdot{}^\circ F}{Btu}$$

Lastly, find the Convection Resistance from the outside of the insulation to the ambient space.

$$R_{convection} = \frac{1}{hA}$$

$$R_{convection} = \frac{1}{\left(1.5\frac{Btu}{hr\cdot ft^2\cdot{}^\circ F}\right)\left[\pi\left(\frac{5.5}{12}ft\right)(1ft)\right]} = 0.46\frac{hr\cdot{}^\circ F}{Btu}$$

Take the sum of all resistances to determine the total resistance.

$$R_t = R_{pipe} + R_{contact} + R_{insulation} + R_{convection}$$

$$R_t = 0\frac{hr\cdot{}^\circ F}{Btu} + 0.055\frac{hr\cdot{}^\circ F}{Btu} + 2.398\frac{hr\cdot{}^\circ F}{Btu} + 0.46\frac{hr\cdot{}^\circ F}{Btu} = 2.91\frac{hr\cdot{}^\circ F}{Btu}$$

Determine the total heat transfer per linear foot of pipe by dividing the ΔT by the total resistance.

$$\dot{Q} = \frac{\Delta T}{R_t}$$

$$\dot{Q} = \frac{195^\circ F - 60^\circ F}{2.91\frac{hr\cdot{}^\circ F}{Btu}} = 15.2\frac{Btu}{hr}\,(for\,1\,linear\,foot) = 46.3\frac{Btu}{hr\cdot ft}$$

Answer B